

# An Improved Algorithm to Remove DC Offsets from Fault Current Signals

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**Abstract**— Fault current signals that are processed by digital relays consist of DC, fundamental, and harmonic components. Filtering algorithms are necessary to eliminate the DC and harmonic components from these signals. Several algorithms have been proposed for this task which vary in their accuracy, response time, and computational burden. The conventional Discrete Fourier Transform (DFT) can eliminate harmonics and is commonly used to estimate the fundamental frequency phasor. But its accuracy is lower as it does not filter the DC offset. Other algorithms including variants of DFT attempt to improve the accuracy and response time. This paper proposes a technique that takes into account the exponential variation of the DC offset and more accurately determines the fundamental component. The effectiveness of this method is evaluated by simulation on a 2-machine system and also compared against existing phasor measurement methods. Simulations confirm that the proposed method can more accurately estimate the fundamental component compared to the existing methods.

**Index Terms**— DC component, fundamental component, estimation

## I. INTRODUCTION

PROTECTION relays require the input voltage and current signals to be free from harmonics and DC components so that fault conditions can be more accurately detected. Filtering algorithms are necessary to extract the fundamental component from input signals that contain harmonics and DC components. The conventional Discrete Fourier Transform (DFT) is most commonly used to extract the fundamental components from the measured waveforms [1-6]. The DFT can eliminate the harmonic components but not the DC component. The DC component is a non-periodic signal that has a large frequency spectrum. It contributes to the overshoot and oscillations that are present in the estimation of the fundamental component when using the DFT giving rise to an error of around 15.1% [2].

Manuscript received 6<sup>th</sup> Nov. 2015, revised 1<sup>st</sup> Apr and 22<sup>nd</sup> Jun, accepted 2<sup>nd</sup> Oct. 2016.

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Several existing digital filtering algorithms estimate the fundamental component by eliminating the DC offset present in the measured current signals [1-7]. In the improved DFT algorithm proposed in [1], the fundamental component is estimated by obtaining the even and odd samples of the DFT for the fault current signal. This algorithm is more accurate than the conventional DFT as it eliminates the DC offset. However, it experiences a slow response time since it only carries out the estimation after one cycle and needs to perform extensive computations. In [2], a new DFT-based phasor estimation method is applied to fault current signals containing harmonics, noise, and DC components. This method uses a few basic mathematical operations to accurately estimate the fundamental frequency component but takes one cycle. A modified DFT-based phasor estimation algorithm was proposed in [3] to eliminate the effect of DC components on the fault current signal. It experiences minimal overshoot and provides a smooth transient response. It is also robust against noise, but takes one cycle to perform the estimation. An online algorithm to remove decaying DC offsets from fault current signals was proposed in [4]. It performs the estimation more accurately and efficiently than the conventional DFT method, but takes one cycle to carry out the estimation. An iterative algorithm proposed in [5] takes four consecutive samples of a sinusoidal input current signal to determine the fundamental and DC components. This method has a fast response time since it estimates and eliminates the DC offset from the fundamental component using just four samples. But it assumes the DC offset to be constant and therefore does not consider the exponential decay of DC offset that commonly occurs in fault current signals. In [6], a modified DFT-based phasor estimation method was proposed to eliminate the DC components from the fault current signal. This method accurately estimates and eliminates the DC component. However, it takes one cycle to perform the estimation and uses extensive computations. It is also sensitive to noise. In [7], an innovative algorithm for estimating and eliminating the DC component from a fault current signal was proposed. This algorithm has faster convergence and better accuracy than the conventional DFT. However, it is computationally more complex as it performs the estimation of the DC component using the Taylor series approximation.

A number of dynamic phasor estimation algorithms have been proposed in [8-11] to estimate the phasor and eliminate both the DC offset and dynamic characteristics. In [8], an adaptive dynamic phasor estimation algorithm based on the modified empirical mode decomposition (EMD) technique was used to improve the accuracy of the phasor estimation.

This technique has high accuracy and convergence speed. However, the combination of the EMD technique with the Hilbert Transform (HT) is unable to completely eliminate the noise and high frequency components. It also takes 2.25 cycles to estimate. A dynamic phasor estimation algorithm for fault currents from DGs was proposed in [9]. This algorithm accurately estimates the decaying amplitude and time constant of the fundamental component. It also distinguishes between the decaying DC and fundamental components but takes more than one cycle to estimate. An adaptive dynamic phasor estimator was proposed in [11] to accurately estimate the phasor under dynamic and DC component conditions. This algorithm is computationally efficient and can be easily implemented in Phasor Measurement Unit (PMU) applications. However, it produces large errors due to the exponentially decaying DC offset and takes more than one cycle to compute. In [10], phasor and frequency estimators are proposed to remove errors due to dynamic characteristics and/or decaying DC components. These algorithms are sensitive to noise and take more than one cycle to estimate.

This paper proposes a digital filtering algorithm that extends the method in [5] by considering the exponential decay of the DC offset as well as the harmonic and sub-harmonic components. A signal averaging filter pre-processes the input signal to eliminate noise. The mathematical basis for the proposed algorithm is presented in this paper. The effectiveness of the algorithm is evaluated by simulation and also compared with the cosine [12] and improved DFT algorithms [1] as well as the iterative method [5].

The rest of this paper is organised as follows: The proposed algorithm is described in Section II. Section III briefly describes a 2-machine model used to test the proposed algorithm. Simulation results that evaluate the effectiveness of the proposed algorithm and also compare it with existing algorithms are presented in Section IV. Section V summarises the paper.

## II. PROPOSED ALGORITHM

The proposed algorithm assumes that the DC offset decays exponentially unlike the iterative algorithm in [5] where the DC offset was assumed to be constant. The proposed algorithm is derived in this section for the case where the time constant is known as well as when it is unknown. The algorithm uses a known time constant to study the effect of the changing DC component parameters on the fundamental component. For processing an input fault current signal from the measurement device, it assumes an unknown time constant. The measurement signals also contain Gaussian noise with zero mean and varying standard deviations.

### A. Algorithm with Known Time Constants

The input signal containing Gaussian noise is first processed by a signal averaging filter to eliminate the noise. This filter samples the entire input signal for a given number of times. All these samples are then summed at all sampling instances and the average of the samples obtained for the entire signal. The output signal from the signal averaging filter is then processed by the proposed algorithm to obtain the fundamental component magnitude. In this derivation of the

algorithm, the time constant is assumed to be known. Fourteen consecutive samples are extracted from the fault current input signal which consists of DC, harmonic, sub-harmonic, and fundamental components. The input fault current signal can be expressed as the following equation:

$$i(t) = A_{dc1} \exp\left(-\frac{(t+k\Delta t)}{\tau_1}\right) + A_{dc2} \exp\left(-\frac{(t+k\Delta t)}{\tau_2}\right) + A_{ac} \sin(2\pi f(t+k\Delta t)) + \sum_{i=1}^5 A_{ac(2i+1)} \sin(2(2i+1)\pi f(t+k\Delta t)) + \sum_{j=1}^5 A_{ac\frac{1}{(2j+1)}} \sin\left(\frac{2}{(2j+1)}\pi f(t+k\Delta t)\right) \quad (1)$$

where  $\Delta t$  is the known sampling period,  $f$  is the known system frequency,  $\tau$  is the time constant of the DC component,  $k$  is the sampling constant that takes on values from 0 to 13. The fundamental frequency value is assumed to be known since it should always match the power system frequency. The unknown values are the amplitudes of the fundamental ( $A_{ac}$ ), harmonic ( $A_{ac(2i+1)}$ ), and sub-harmonic ( $A_{ac\frac{1}{(2j+1)}}$ ) components, DC component amplitudes of  $A_{dc1}$  and  $A_{dc2}$ , and continuous time  $t$ .

To discretise the general equation (1), let  $t = n\Delta t$  and  $\Delta t = \frac{1}{Nf}$  and thus  $t = \frac{n}{Nf}$  where  $n$  is the sample number and  $N$  is the number of samples in a cycle. To simplify the equation further, let  $\alpha_1 = \exp\left(-\frac{\Delta t}{\tau_1}\right)$  and  $\alpha_2 = \exp\left(-\frac{\Delta t}{\tau_2}\right)$ . The resulting equation is as follows:

$$i(n) = A_{dc1}\alpha_1^{n+k} + A_{dc2}\alpha_2^{n+k} + A_{ac} \sin\left(\frac{2\pi}{N}(n+k)\right) + \sum_{i=1}^5 A_{ac(2i+1)} \sin\left(\frac{2(2i+1)\pi}{N}(n+k)\right) + \sum_{j=1}^5 A_{ac\frac{1}{(2j+1)}} \sin\left(\frac{2\pi}{(2j+1)N}(n+k)\right) \quad (2)$$

The unknown values are  $A_{dc1}$ ,  $A_{dc2}$ ,  $A_{ac}$ , sample number  $n$ , as well as the amplitudes of the third, fifth, seventh, ninth, and eleventh harmonics and sub-harmonics. The known values are  $\alpha_1$ ,  $\alpha_2$ ,  $N$ , and  $k$ . The fourteen nonlinear equations were solved simultaneously using MATLAB's fsolve function to obtain the unknown values. A window of fourteen consecutive time samples is moved across the time scale until the end of the time interval to improve the estimation accuracy.

### B. Algorithm with Unknown Time Constants

The measurement signal that also contains noise would be processed by a Gaussian pulse-shaping filter to remove the noise before being input to the proposed algorithm. In the Gaussian pulse-shaping filter, the Gaussian function is convolved with the measurement signal. The derivation of the proposed algorithm is given below when the time constant  $\tau$  is unknown. Harmonic, sub-harmonic, and additional DC components are not included as they would make the following derivation rather unwieldy. Four samples are extracted from the fault current input signal which consists of both DC and fundamental components. They are represented

by the following equations:

$$A = V_m \cos(2\pi f t) + A_{dc} \exp\left(-\frac{t}{\tau}\right) \quad (3)$$

$$B = V_m \cos(2\pi f(t + \Delta t)) + A_{dc} \exp\left(-\frac{(t + \Delta t)}{\tau}\right) \quad (4)$$

$$C = V_m \cos(2\pi f(t + 2\Delta t)) + A_{dc} \exp\left(-\frac{(t + 2\Delta t)}{\tau}\right) \quad (5)$$

$$D = V_m \cos(2\pi f(t + 3\Delta t)) + A_{dc} \exp\left(-\frac{(t + 3\Delta t)}{\tau}\right) \quad (6)$$

where  $A$ ,  $B$ ,  $C$ , and  $D$  are the measured samples,  $\Delta t$  is the known sampling period,  $f$  is the known system frequency, and  $\tau$  is the time constant of the DC component. The unknown values are the fundamental component amplitude  $V_m$ , DC component amplitude  $A_{dc}$ , and continuous time  $t$ .

Let  $t = n\Delta t$  and  $\Delta t = \frac{1}{Nf}$  and thus  $t = \frac{n}{Nf}$  where  $n$  is the sample number and  $N$  is the number of samples in a cycle. Let  $\alpha = \exp\left(-\frac{\Delta t}{\tau}\right)$ . Equations (3) - (6) can be transformed as follows:

$$A = V_m \cos\left(\frac{2\pi n}{N}\right) + A_{dc} \alpha^n \quad (7)$$

$$B = V_m \cos\left(\frac{2\pi}{N}(n + 1)\right) + A_{dc} \alpha^{n+1} \quad (8)$$

$$C = V_m \cos\left(\frac{2\pi}{N}(n + 2)\right) + A_{dc} \alpha^{n+2} \quad (9)$$

$$D = V_m \cos\left(\frac{2\pi}{N}(n + 3)\right) + A_{dc} \alpha^{n+3} \quad (10)$$

Rearranging (7) - (10) using trigonometric identities to eliminate  $A_{dc}$  results in the following equations:

$$D - \alpha C = V_m \left( \cos\left(\frac{2\pi n}{N}\right) \left( \cos\left(\frac{6\pi}{N}\right) - \alpha \cos\left(\frac{4\pi}{N}\right) \right) - \sin\left(\frac{2\pi n}{N}\right) \left( \sin\left(\frac{6\pi}{N}\right) - \alpha \sin\left(\frac{4\pi}{N}\right) \right) \right) \quad (11)$$

$$C - \alpha B = V_m \left( \cos\left(\frac{2\pi n}{N}\right) \left( \cos\left(\frac{4\pi}{N}\right) - \alpha \cos\left(\frac{2\pi}{N}\right) \right) - \sin\left(\frac{2\pi n}{N}\right) \left( \sin\left(\frac{4\pi}{N}\right) - \alpha \sin\left(\frac{2\pi}{N}\right) \right) \right) \quad (12)$$

$$B - \alpha A = V_m \left( \cos\left(\frac{2\pi n}{N}\right) \left( \cos\left(\frac{2\pi}{N}\right) - \alpha \right) - \sin\left(\frac{2\pi n}{N}\right) \sin\left(\frac{2\pi}{N}\right) \right) \quad (13)$$

By dividing (11) with (12) and (12) with (13) to eliminate  $V_m$ , the following equations are obtained:

$$\frac{D - \alpha C}{C - \alpha B} = \frac{\cos\left(\frac{2\pi n}{N}\right) \left( \cos\left(\frac{6\pi}{N}\right) - \alpha \cos\left(\frac{4\pi}{N}\right) \right) - \sin\left(\frac{2\pi n}{N}\right) \left( \sin\left(\frac{6\pi}{N}\right) - \alpha \sin\left(\frac{4\pi}{N}\right) \right)}{\cos\left(\frac{2\pi n}{N}\right) \left( \cos\left(\frac{4\pi}{N}\right) - \alpha \cos\left(\frac{2\pi}{N}\right) \right) - \sin\left(\frac{2\pi n}{N}\right) \left( \sin\left(\frac{4\pi}{N}\right) - \alpha \sin\left(\frac{2\pi}{N}\right) \right)} \quad (14)$$

$$\frac{C - \alpha B}{B - \alpha A} = \frac{\cos\left(\frac{2\pi n}{N}\right) \left( \cos\left(\frac{4\pi}{N}\right) - \alpha \cos\left(\frac{2\pi}{N}\right) \right) - \sin\left(\frac{2\pi n}{N}\right) \left( \sin\left(\frac{4\pi}{N}\right) - \alpha \sin\left(\frac{2\pi}{N}\right) \right)}{\cos\left(\frac{2\pi n}{N}\right) \left( \cos\left(\frac{2\pi}{N}\right) - \alpha \right) - \sin\left(\frac{2\pi n}{N}\right) \sin\left(\frac{2\pi}{N}\right)} \quad (15)$$

To eliminate  $n$ , rearrange (14) and (15) and divide both sides by  $\cos\left(\frac{2\pi n}{N}\right)$  to yield the following equations:

$$\begin{aligned} \tan\left(\frac{2\pi n}{N}\right) &= \frac{(D - \alpha C) \left( \cos\left(\frac{4\pi}{N}\right) - \alpha \cos\left(\frac{2\pi}{N}\right) \right) - (C - \alpha B) \left( \cos\left(\frac{6\pi}{N}\right) - \alpha \cos\left(\frac{4\pi}{N}\right) \right)}{(D - \alpha C) \left( \sin\left(\frac{4\pi}{N}\right) - \alpha \sin\left(\frac{2\pi}{N}\right) \right) - (C - \alpha B) \left( \sin\left(\frac{6\pi}{N}\right) - \alpha \sin\left(\frac{4\pi}{N}\right) \right)} \end{aligned} \quad (16)$$

$$\begin{aligned} \tan\left(\frac{2\pi n}{N}\right) &= \frac{(C - \alpha B) \left( \cos\left(\frac{2\pi}{N}\right) - \alpha \right) - (B - \alpha A) \left( \cos\left(\frac{4\pi}{N}\right) - \alpha \cos\left(\frac{2\pi}{N}\right) \right)}{(C - \alpha B) \sin\left(\frac{2\pi}{N}\right) - (B - \alpha A) \left( \sin\left(\frac{4\pi}{N}\right) - \alpha \sin\left(\frac{2\pi}{N}\right) \right)} \end{aligned} \quad (17)$$

By equating (16) and (17), rearranging and simplifying the resulting equation, the following expression for  $\alpha$  is obtained as in (18):

$$\alpha = \frac{D + B - 2C \cos\left(\frac{2\pi}{N}\right)}{A + C - 2B \cos\left(\frac{2\pi}{N}\right)} \quad (18)$$

In this improved algorithm, four consecutive samples of  $A$ ,  $B$ ,  $C$ , and  $D$  are obtained from the measurement devices. The  $\alpha$  value is then determined from (18) using the known values of  $A$ ,  $B$ ,  $C$ ,  $D$ , and  $N$ . The values of  $A$ ,  $B$ ,  $C$ ,  $D$ ,  $N$ , and  $\alpha$  are substituted into (16) or (17). The known values of  $n$ ,  $N$ ,  $\alpha$ ,  $A$ ,  $B$ ,  $C$ , and  $D$  are substituted into equations (7) - (10). These equations are then solved simultaneously to obtain the unknown values  $V_m$  and  $A_{dc}$  as given below.

$$\begin{bmatrix} V_m \\ A_{dc} \end{bmatrix} = \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} \backslash \begin{bmatrix} \cos\left(\frac{2\pi n}{N}\right) & \alpha^n \\ \cos\left(\frac{2\pi}{N}(n + 1)\right) & \alpha^{n+1} \\ \cos\left(\frac{2\pi}{N}(n + 2)\right) & \alpha^{n+2} \\ \cos\left(\frac{2\pi}{N}(n + 3)\right) & \alpha^{n+3} \end{bmatrix} \quad (19)$$

### III. TWO-MACHINE POWER SYSTEM MODEL DESCRIPTION

The model used to evaluate the effectiveness of the proposed phasor measurement algorithm is a Two-machine system as shown in Fig. 1 and adapted from [5]. It consists of a synchronous generator connected to one end of the transmission line and an infinite bus at the other end. The synchronous generator is modelled as a diesel generator whereas the infinite bus is represented as a three-phase sinusoidal source. Static loads are connected to both ends of the transmission line. The parameters for the sources, transmission line, and loads are defined in Tables I and II.

TABLE I  
PARAMETERS FOR THE SOURCES IN THE TWO-MACHINE SYSTEM

Source	Rated RMS L-N Voltage (kV)	Rated RMS Line Current (kA)	Initial Phase (rad)
Synchronous Generator (SG)	7.967	5.02	3.14
Infinite Source (IS)	7.9617		2.50

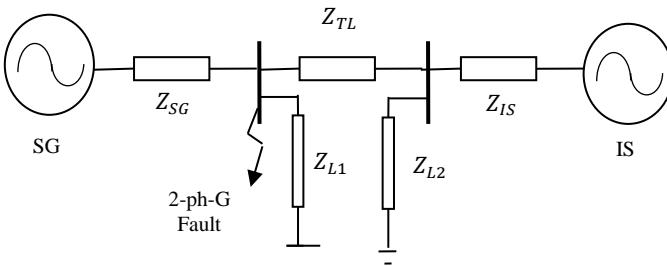


Fig. 1. Diagram of Two-machine test system.

TABLE II  
PARAMETERS FOR THE IMPEDANCES IN THE 2-MACHINE SYSTEM

Impedance	Resistance ( $\Omega$ )	Reactance (H)	Connection
SG Impedance ( $Z_{SG}$ )	1.02	0.105	R-L
IS Impedance ( $Z_{IS}$ )	1.0	0.1	R-L
Transmission ( $Z_{TL}$ )	0.91	0.0052	R-L
Load 1 ( $Z_{L1}$ )	15		
Load 2 ( $Z_{L2}$ )	1.8		

## IV. SIMULATION RESULTS

Simulations were performed in MATLAB to evaluate the effectiveness of the proposed algorithm and compare it to the other existing algorithms. First the proposed algorithm that assumes a known time constant was evaluated using the test signal,

$$i(t) = A_{dc1} \exp\left(-\frac{t}{\tau_1}\right) + A_{dc2} \exp\left(-\frac{t}{\tau_2}\right) + \sin(2\pi(50)t) + 2 \sin(6\pi(50)t) + 3 \sin\left(\frac{2}{3}\pi(50)t\right) + 13 \sin(10\pi(50)t) + 5 \sin\left(\frac{2}{5}\pi(50)t\right) + 9 \sin(14\pi(50)t) + 3 \sin\left(\frac{2}{7}\pi(50)t\right) + 11 \sin(18\pi(50)t) + 4 \sin\left(\frac{2}{9}\pi(50)t\right) + 11 \sin(22\pi(50)t) + 6 \sin\left(\frac{2}{11}\pi(50)t\right) + \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{z^2}{2\sigma^2}\right)$$

where  $A_{dc1}$  and  $A_{dc2}$  are the amplitudes of the exponentially decaying DC components;  $\tau_1$  and  $\tau_2$  are the exponential time constants; and  $\sigma$  and  $z$  are the standard deviation and Gaussian random variable. The sampling frequency was set at 5 kHz and system frequency as 50 Hz.

The proposed algorithm was compared with the cosine [12] and improved DFT methods [1] as well as the iterative method [5]. In the improved DFT method in [1], the fundamental component is estimated by obtaining even and odd samples for both the AC and DC components of the conventional DFT for the fault current signal. The odd samples are subtracted from the even samples for the AC and DC components. The AC and DC terms are summed and then their amplitude is calculated before the AC component magnitude is determined by the following equation:

$$A_1 \approx \frac{|A_{est}^e(n)|}{2 \sin\left(\frac{\pi}{N}\right)}$$

The proposed algorithm with the unknown time constant as

described in Section II.B was evaluated using MATLAB Simulink on the 2-machine system model of Section III. when a 2-ph-G fault occurs on bus 1. Measurements of the input current signal were made on the faulted bus so that the DC and fundamental components of the signal could be computed by the algorithm. Gaussian noise with zero mean and standard deviations of 0.1, 1.0, and 2.0 was added to the measurement signal on bus 1.

## A. Comparison with Existing Algorithms

Fig. 2 shows the output of the fundamental component by the proposed algorithm along with those of the cosine, improved DFT, and iterative algorithms. The proposed algorithm estimates the fundamental component as 1pu after the time required for fourteen samples at the beginning of the first cycle. The estimation using the cosine method gives 1pu after 1.25 cycles. On the other hand, the improved DFT gives the estimate as 1pu after one cycle. The estimate by the iterative method is 49.5529pu after four samples. Compared to the cosine and improved DFT methods, the proposed algorithm is faster since it produces the output after fourteen samples at the beginning of the first cycle. The iterative algorithm requires only four samples but it is less accurate.

## B. Variations in Time Constants, DC Offset Magnitudes, and Standard Deviations of the Gaussian Noise

The effect of the variation in the time constants  $\tau_1$  and  $\tau_2$ , DC offset magnitudes  $A_{dc1}$  and  $A_{dc2}$ , and standard deviation of the Gaussian noise  $\sigma$  on the estimation of the fundamental component magnitude was evaluated by simulation. For  $A_{dc1}$  ( $A_{dc2}$ ), the values of 0.5 (0.7), 1.0 (1.2), and 1.5 (1.7) were used to evaluate the effect of small variations on the estimation of fundamental component magnitude. The values of  $\tau_1$  ( $\tau_2$ ) were chosen as 0.5 (0.7), 1.0 (1.2), and 1.5 (1.7) so that the incremental effect of the DC decaying time on the fundamental component magnitude can be evaluated. The  $\sigma$  values of 0.1, 1.0, and 2.0 were used to evaluate the effect of small changes in the standard deviation of the Gaussian noise.

Figs. 3 and 4 show the fundamental components from the different phasor measurement algorithms for  $\tau_1$  ( $\tau_2$ ) values of 1.0 (1.2) and 1.5 (1.7) respectively assuming that  $A_{dc1} = 0.5$  ( $A_{dc2} = 0.7$ ) and  $\sigma = 0.1$  are fixed. Figs. 5 and 6 show the results from various algorithms for  $A_{dc1}$  ( $A_{dc2}$ ) values of 1.0 (1.2) and 1.5 (1.7) respectively when  $\tau_1 = 0.5$  ( $\tau_2 = 0.7$ ) and  $\sigma = 0.1$  are unchanged. The fundamental components from various algorithms are shown in Figs. 7 and 8 for  $\sigma$  values of 0.1, 1.0, and 2.0 for fixed values of  $\tau_1 = 0.5$  ( $\tau_2 = 0.7$ ) and  $A_{dc1} = 0.5$  ( $A_{dc2} = 0.7$ ).

For the proposed algorithm, the fundamental component output remains at 1pu for all values of  $\tau_1$  ( $\tau_2$ ),  $A_{dc1}$  ( $A_{dc2}$ ), and  $\sigma$  that were considered. The outputs of the iterative algorithm are 47.6060pu and 45.3444pu for  $\tau_1$  ( $\tau_2$ ) values of 1.0 (1.2) and 1.5 (1.7) respectively; for  $A_{dc1}$  ( $A_{dc2}$ ) values of 1.0 (1.2) and 1.5 (1.7), the outputs are 47.9354pu and 50.3343pu respectively; for  $\sigma$  values of 1.0 and 2.0, the outputs are 345.8030pu and 45.4698pu respectively. The iterative algorithm does not consider the exponentially varying DC offsets, harmonic and sub-harmonic components, or the noise.

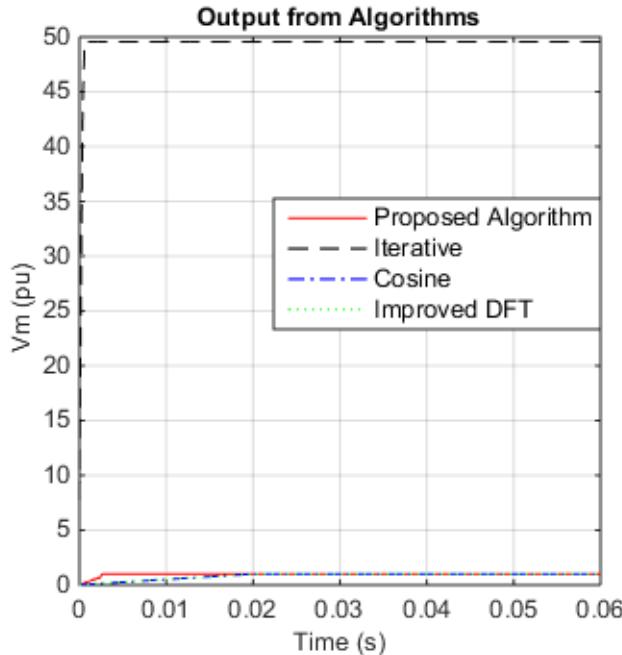


Fig. 2. Fundamental component measurement for the different phasor measurement algorithms for  $\tau_1 = 0.5$ ,  $\tau_2 = 0.7$ ,  $A_{dc1} = 0.5$ ,  $A_{dc2} = 0.7$ , and  $\sigma = 0.1$ .

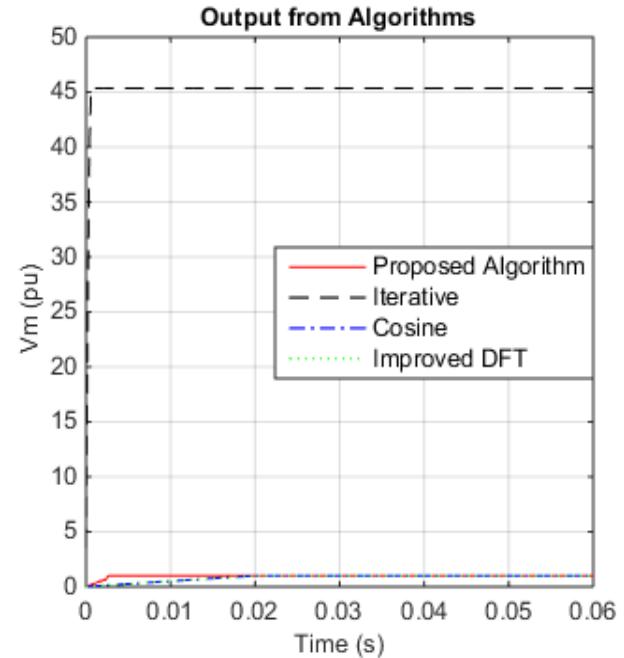


Fig. 4. Fundamental component measurement for the different phasor measurement algorithms for  $\tau_1 = 1.5$ ,  $\tau_2 = 1.7$ ,  $A_{dc1} = 0.5$ ,  $A_{dc2} = 0.7$ , and  $\sigma = 0.1$ .

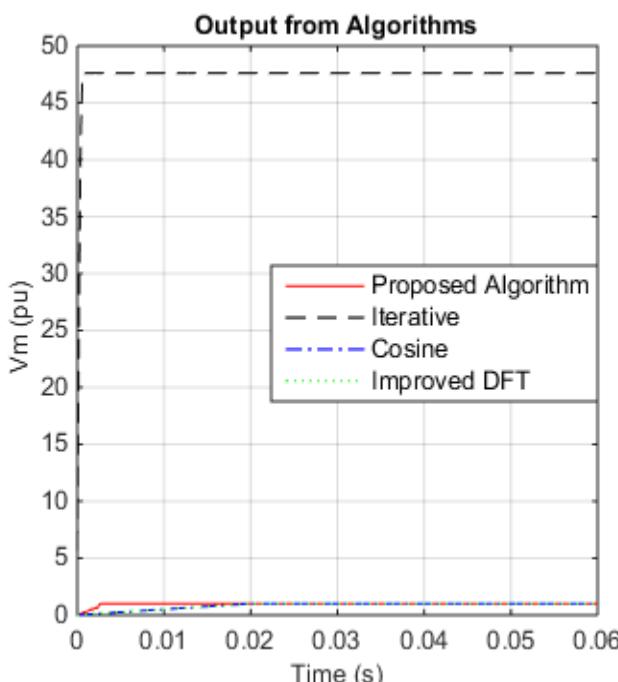


Fig. 3. Fundamental component measurement for the different phasor measurement algorithms for  $\tau_1 = 1$ ,  $\tau_2 = 1.2$ ,  $A_{dc1} = 0.5$ ,  $A_{dc2} = 0.7$ , and  $\sigma = 0.1$ .

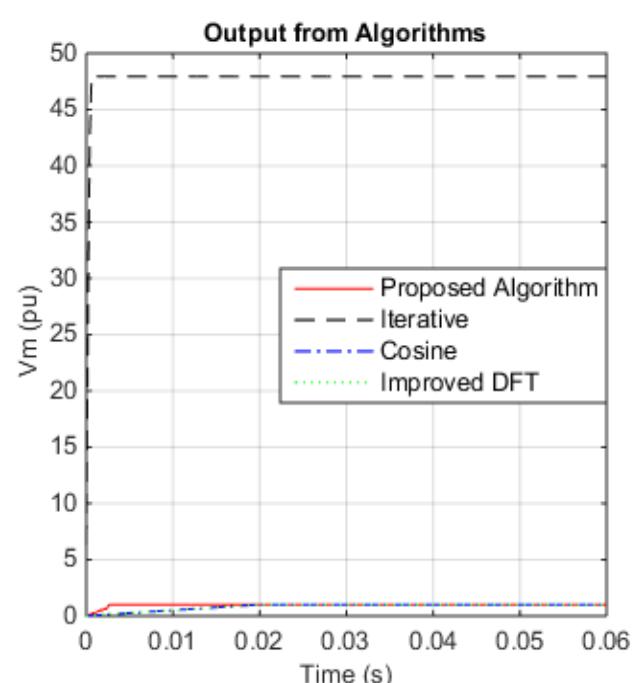


Fig. 5. Fundamental component measurement for the different phasor measurement algorithms for  $\tau_1 = 0.5$ ,  $\tau_2 = 0.7$ ,  $A_{dc1} = 1$ ,  $A_{dc2} = 1.2$ , and  $\sigma = 0.1$ .

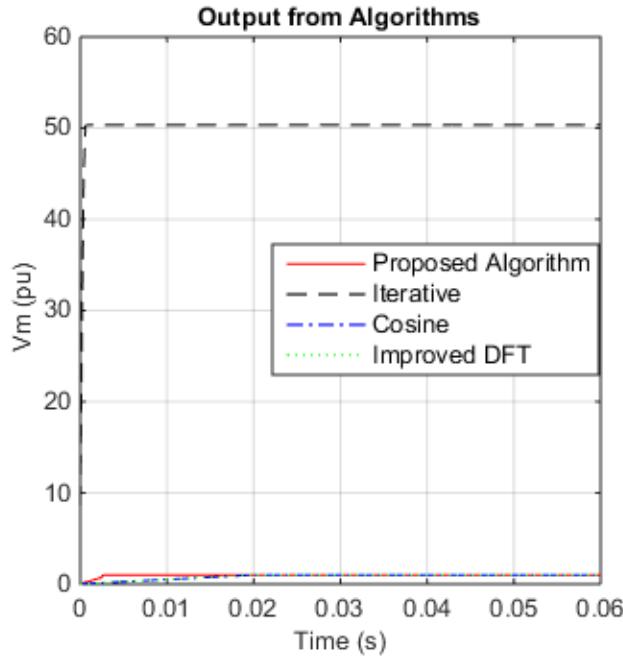


Fig. 6. Fundamental component measurement for the different phasor measurement algorithms for  $\tau_1 = 0.5$ ,  $\tau_2 = 0.7$ ,  $A_{dc1} = 1.5$ ,  $A_{dc2} = 1.7$ , and  $\sigma = 0.1$ .

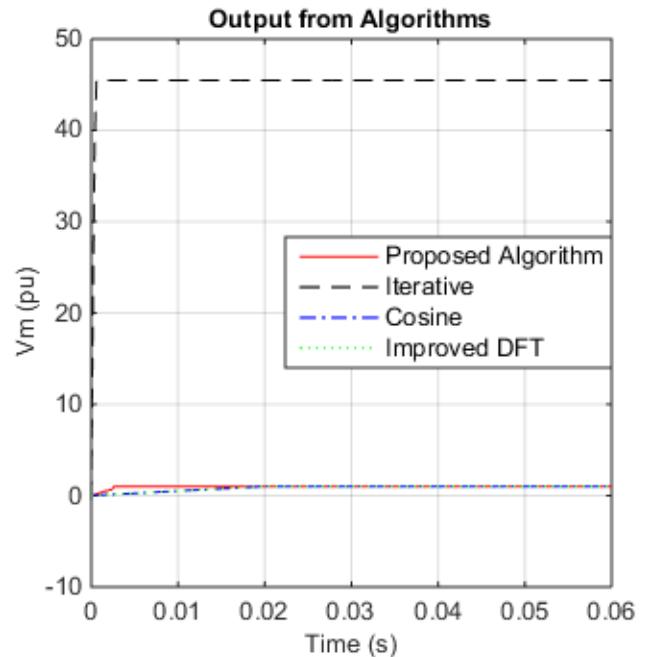


Fig. 8. Fundamental component measurement for the different phasor measurement algorithms for  $\tau_1 = 0.5$ ,  $\tau_2 = 0.7$ ,  $A_{dc1} = 0.5$ ,  $A_{dc2} = 0.7$ , and  $\sigma = 2.0$ .

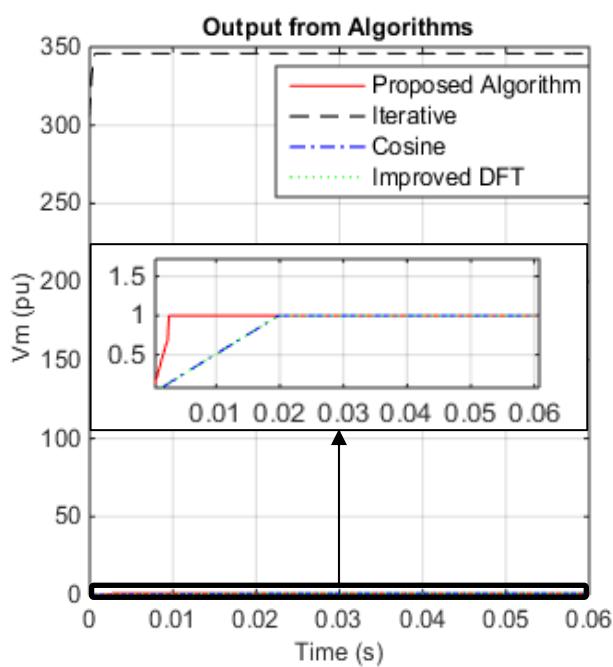


Fig. 7. Fundamental component measurement for the different phasor measurement algorithms for  $\tau_1 = 0.5$ ,  $\tau_2 = 0.7$ ,  $A_{dc1} = 0.5$ ,  $A_{dc2} = 0.7$ , and  $\sigma = 1.0$ .

Both the cosine and improved DFT methods assume that the harmonic and sub-harmonic components are filtered before extracting the fundamental component. These two methods only deal with one exponentially decaying DC component. For the cosine method, the output fluctuates between 0.9999-1.0001pu for  $A_{dc1}$  ( $A_{dc2}$ ) values of 1.0 (1.2) and 1.5 (1.7) and remains close to 1pu for the other cases considered above. In the improved DFT method, the fundamental component output is close to 1pu since the exponential term has a much lower value compared to the conventional DFT method. The proposed algorithm estimates the fundamental component magnitude more accurately compared to the other methods for all values of  $\tau_1$ ,  $\tau_2$ ,  $A_{dc1}$ ,  $A_{dc2}$ , and  $\sigma$  considered in this evaluation.

### C. Simulation of Proposed Algorithm on 2-Machine System

The proposed algorithm in Section II.B that assumes the time constant to be unknown was simulated on bus 1 in the 2-machine system to evaluate its effectiveness in separating the fundamental and DC components of the fault current. Fig. 9 shows the input current signals where phases A and B increases during the fault. The fundamental and DC components are shown in Fig. 10 when  $\sigma$  is 0.1. The DC component for phase B increases and decays steadily at the start and end of the fault. On the other hand, at the beginning and end of the fault, the phase A DC component is shown as decreasing and then increasing steadily until it is zero since phase A lags phase B by  $120^\circ$ .

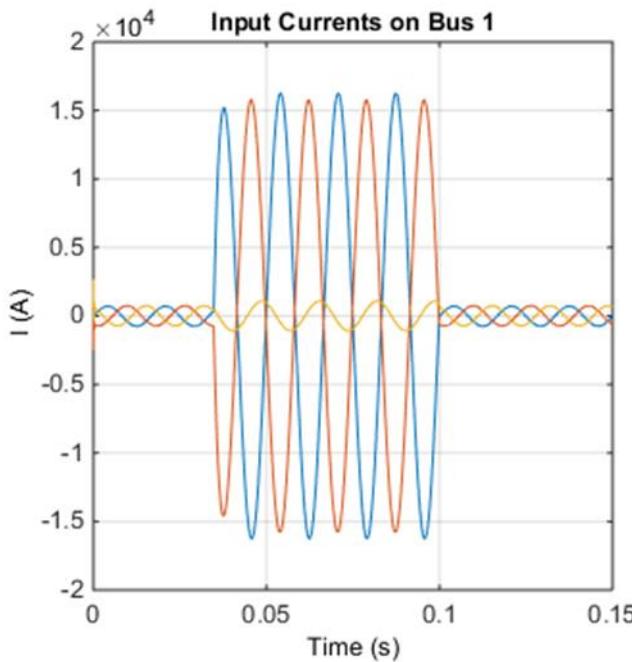


Fig. 9. Input current signal on Bus 1 in Two-machine system.

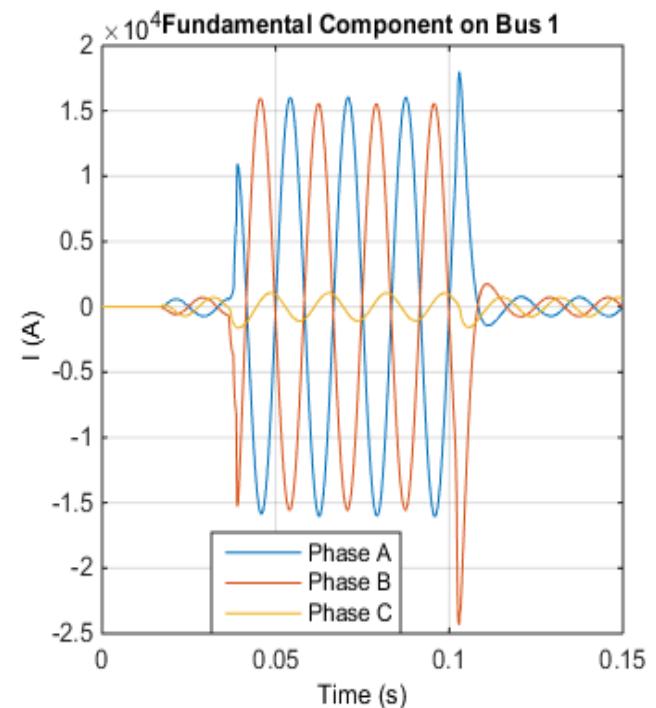


Fig. 11. Fundamental component for current signal on Bus 1 in Two-machine system.

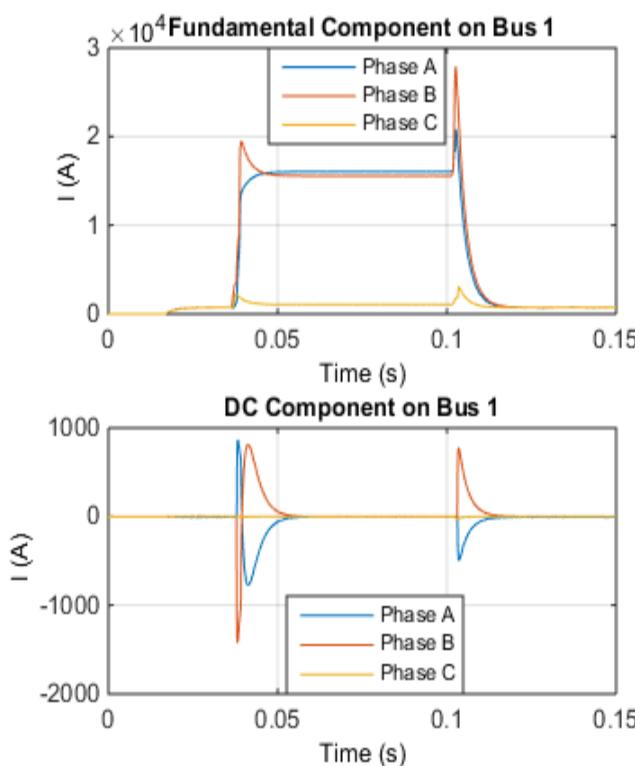


Fig. 10. Fundamental and DC components for the current signal on Bus 1 in Two-machine system for the improved method.

The fundamental component for phases A and B have smoothly increasing magnitudes at the beginning of the fault, stays constant during the fault, and decreases at the end of the fault following a spike. Hence the proposed algorithm can effectively separate out the fundamental and DC components from the input current signals. Fig. 11 shows the fundamental frequency component of the input signal in sinusoidal form when  $\sigma$  is 0.1. This output waveform is very close to the input waveform in Fig. 9 since the peak of the DC component shown in Fig. 10 is only about 13% of the fundamental frequency component magnitude. However, the estimated fundamental component values at 0.038s and 0.102s are different to the estimated values between these times. This is because the four consecutive samples were taken during the transitions between faulted and normal conditions. The results for  $\sigma$  values of 1.0 and 2.0 are not shown since they are similar to the case of  $\sigma = 0.1$ .

#### D. Discussions

The proposed algorithm is more accurate compared to the other methods in estimating the fundamental component amplitude. The estimate by the proposed algorithm is unaffected by changes in time constants, DC offset magnitudes, harmonic and sub-harmonic magnitudes, as well as varying standard deviations of Gaussian noise. The iterative algorithm is less accurate since it does not deal with the exponentially varying DC offsets, harmonic and sub-harmonic components, or the Gaussian noise. The improved DFT and cosine methods estimate the fundamental component

magnitude with only small fluctuations from the correct value as they reduce the effect of the DC offset and are unaffected by harmonics, sub-harmonics, and noise.

The iterative and proposed algorithms have faster response times compared to the cosine and improved DFT methods. The proposed algorithm also accurately estimated the DC offset and fundamental component of the input fault current when simulated in a 2-machine system.

## V. CONCLUSIONS

This paper proposed a new algorithm that more accurately estimates the fundamental component of fault current signals by taking fourteen samples at the beginning of the first cycle. Unlike the iterative algorithm, it correctly deals with the exponentially varying DC components, harmonics, subharmonics, and noise in the input current signal. The proposed algorithm was evaluated against the cosine and improved DFT methods as well as the iterative algorithm by simulation. The simulation results showed that the proposed algorithm is more accurate than the iterative method. The cosine and improved DFT methods are nearly as accurate but they require a cycle or more to produce the output. The proposed algorithm can also accurately estimate the fundamental component amplitude even when there are variations in the time constant and DC offset amplitude. It is significantly faster than the existing dynamic phasor estimation algorithms which take more than one cycle to estimate. The algorithm proposed in this paper will be useful in digital relaying schemes since it can accurately and quickly estimate the fundamental component for detecting and clearing fault conditions that occur in power systems.

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