



# Optimal placement of DG in radial distribution systems based on new voltage stability index under load growth



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## ABSTRACT

This paper presents a comparison of Novel Power Loss Sensitivity, Power Stability Index (PSI), and proposed voltage stability index (VSI) methods for optimal location and sizing of distributed generation (DG) in radial distribution network. The main contribution of the paper is: (i) optimal placement of DGs based on Novel Power Loss Sensitivity and PSI methods, (ii) proposed voltage stability index method for optimal DG placement, (iii) comparison of sensitivity methods for DG location and their size calculations, (iv) optimal placement of DG in the presence of load growth, (v) impact of DG placement at combined load power factor, (vii) impact of DG on voltage stability margin improvement. Voltage profile, the real and reactive powers intake by the grid, real and reactive power flow patterns, cost of energy losses, savings in cost of energy loss and cost of power obtained from DGs are determined. The results show the importance of installing the suitable size of DG at the suitable location. The results are obtained with all sensitivity based methods on the IEEE 12-bus, modified 12-bus, 69-bus and 85-bus test systems.

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## Introduction

The Electric Power Research Institute defines distributed generation as generation from 'a few kilowatts up to 50 MW' [1]. The International Conference on Large High Voltage Electric Systems (CIGRÉ) defines DG as 'smaller than 50–100 MW' [2]. International Energy Agency (IEA) defines distributed generation (DG) as generating plant serving a customer on-site or providing support to a distribution network, connected to the grid at distributed level voltages. Renewable energy based DG is developing fast all over the world in recent years due to its promising potential to reduce the portion of fossil energy consumption in electric power generation and mitigate power losses and harmful carbon emissions [3,4]. The impact of DG on radial distribution network i.e. voltage support, loss reduction, and distribution capacity release, power quality issues and environmental benefits is explained in [5,6].

A new combined algorithm based on GA & PSO is presented in [7] to evaluate the DG site and size in distribution network. A multi-objective index-based approach for optimally determining the size and location of multi-distributed generation units in distribution systems with different load models is proposed in [8] using PSO. In [9] the optimal placement of different types of DGs has been proposed based on particle swarm optimization (PSO)

technique. Optimal size and location of multiple DGs are found using particle swarm optimization to minimization of power loss without violating system constraints considering load growth [10]. A new long term scheduling is presented in [11] for optimal allocation and sizing of different types of DG units in the distribution networks in order to minimize power losses using analytical and PSO algorithm. Authors presented optimal DG allocation and sizing in distribution systems with an objective of loss minimization, guarantee acceptable reliability level and voltage profile using GA [12]. A goal programming technique is developed in [13] for formulation and evaluation of a multi objective function, for optimal planning of DG units in the distribution system. Multiple DG placement using improved analytical method and loss sensitivity factor (LSF) method is presented in [14]. A modified voltage index method is proposed [15] to place and size the DG units to improve the voltage stability margin, without violating system constraints using mixed-integer nonlinear programming. In [16], a multi-objective framework as a nonlinear programming (NLP) is proposed for optimal placement and sizing of DG units. Objective functions include minimizing the number of DGs and power losses as well as maximizing voltage stability margin. Exact loss formula is used [17] to find optimum sizing for DG in each buses to minimize the total real power loss based on the voltage stability index (SI). A novel index is developed considering stable node voltages referred as power stability index (PSI) [18]. The PSI is used to identify the most critical bus in the system that can lead to system

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voltage instability when load increase above certain limit. The DG is placed at the most sensitive bus. A methodology for the integration of dispatchable and non-dispatchable renewable DG units for minimizing annual energy losses is presented in [19] using analytical expressions. Cuckoo search (CS) algorithm and a constructive heuristic algorithm for solving the problem of DG allocation is presented in [20,21]. A multi-objective function to determine the optimal locations to place DGs in distribution system to minimize power loss of the system and enhance reliability improvement and voltage profile is presented in [22] based on dynamic programming. The problem of optimal placement, including size, is formulated for two different objectives, namely, social welfare maximization and profit maximization. The candidate locations for DG placement are identified on the basis of locational marginal price (LMP) [23]. An analytical expression based on real power loss sensitivity to calculate optimal DG size and optimal location of DG minimizing power losses in a distribution network was proposed in [24]. Optimal sizing and sitting decisions for DG capacity planning using heuristic approach was proposed in [25]. A simple conventional iterative search technique along with Newton Raphson method of load flow study is implemented for DG sizing and location with an objective to lower down both cost and loss very effectively. The paper also focuses on optimization of weighting factor, which balances the cost and the loss factors [26]. A comparison of sensitivity based approaches for optimal location and sizing of distributed generation in a distribution network is presented in [27]. Calculation of cost of DG is given in [29] based on conventional, triangular, and complex power limit.

In this paper work, a new voltage stability index (VSI) is developed for optimal placement of DG in radial distribution systems. After identifying the candidate bus for DG placement, the search technique is used to determine optimal size of DG to minimize total power loss. Also, the proposed VSI method is compared with other existing sensitivity based methods i.e., novel power loss and PSI approaches for optimal allocation of DG. In this paper, operation of the DG at unity, 0.9 lagging and combined load power factors are considered. Voltage stability margin (VSM) values are computed for distribution network with and without installation of DGs. The load growth factor has also been considered in this study which is essential for the planning and expansion of the existing systems. The cost of energy loss, cost of loss savings and cost of power supplied from DGs are also calculated and comparison has been provided. The results have been obtained on 12-bus [34], modified 12-bus, 69-bus [35] and 85-bus [30–33] systems.

### Proposed voltage stability index for optimal DG placement in radial distribution system

A simple radial distribution system (RDS) with source at one end and load at the other end with two nodes is shown in Fig. 1. The mathematical model of the proposed voltage stability index is delineated below:

The branch current  $I_{12}$  can be calculated using Eq. (1).

$$I_{12} = \left[ \frac{P_2 + jQ_2}{V_2 \angle \delta} \right]^* \quad (1)$$

The receiving end bus voltage can be written as:

$$V_2 \angle \delta = V_1 \angle 0 - (R + jX)I_{12} \quad (2)$$

Substitute Eq. (1) in Eq. (2),

$$V_2 \angle \delta = V_1 \angle 0 - (R + jX) \left[ \frac{P_2 + jQ_2}{V_2 \angle \delta} \right]^* \quad (3)$$

$$V_2 \angle \delta = V_1 \angle 0 - (R + jX) \left[ \frac{P_2 - jQ_2}{V_2 \angle -\delta} \right] \quad (4)$$

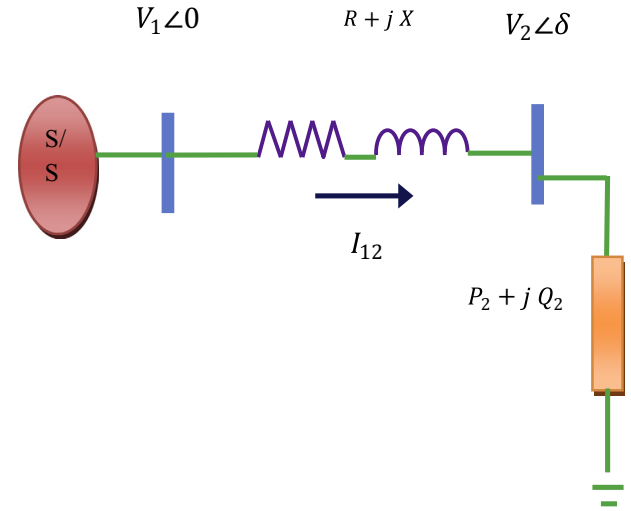


Fig. 1. Equivalent circuit model of RDS.

$$V_2^2 = V_1 V_2 \angle -\delta - (R + jX)(P_2 - jQ_2) \quad (5)$$

$$V_2^2 = V_1 V_2 \cos \delta - jV_1 V_2 \sin \delta - (R + jX)(P_2 - jQ_2) \quad (6)$$

$$V_2^2 + [P_2 R + Q_2 X + j(P_2 X - Q_2 R)] = V_1 V_2 \cos \delta - jV_1 V_2 \sin \delta \quad (7)$$

Separate real and imaginary parts in Eq. (7).

$$V_2^2 + P_2 R + Q_2 X = V_1 V_2 \cos \delta \quad (8)$$

$$P_2 X - Q_2 R = -V_1 V_2 \sin \delta \quad (9)$$

Let  $\delta \approx 0$

$$V_2^2 + P_2 R + Q_2 X = V_1 V_2 \quad (10)$$

$$P_2 X - Q_2 R = 0 \quad (11)$$

$$R = \frac{P_2 X}{Q_2} \quad (12)$$

Substitute Eq. (12) in Eq. (10),

$$V_2^2 + P_2 \frac{P_2 X}{Q_2} + Q_2 X = V_1 V_2 \quad (13)$$

$$V_2^2 - V_2 V_1 + \left( \frac{P_2^2}{Q_2} + Q_2 \right) X = 0 \quad (14)$$

For stable bus voltages,  $b^2 - 4ac \geq 0$ . The new stability index called as voltage stability index (VSI) given by Eq. (17).

$$V_1^2 - 4 \left( \frac{P_2^2}{Q_2} + Q_2 \right) X \geq 0 \quad (15)$$

$$1 \geq \frac{4X}{V_1^2} \left( \frac{P_2^2}{Q_2} + Q_2 \right) \quad (16)$$

$$VSI = \frac{4X}{V_1^2} \left( \frac{P_2^2}{Q_2} + Q_2 \right) \leq 1 \quad (17)$$

Under normal operating conditions, VSI value should be less than unity. If the value of VSI is closer to zero, then the system will be more stable. If the value of VSI is high, then the system is vulnerable to stability. The bus with high VSI value is more sensitivity and it is

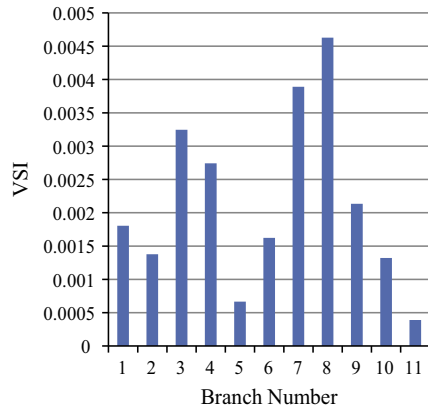


Fig. 2. VSI profile for IEEE 12 bus RDS.

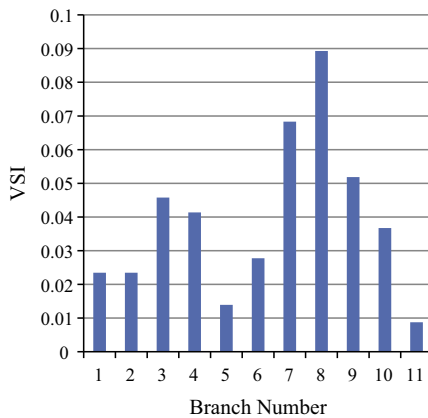


Fig. 3. VSI profile for modified IEEE 12 bus RDS.

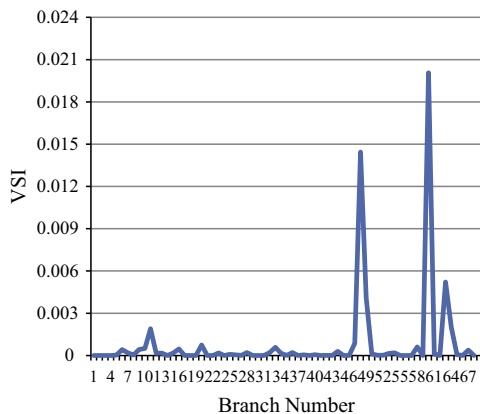


Fig. 4. VSI profile for IEEE 69 bus RDS.

selected for optimal DG placement. The voltage stability index profile is determined for IEEE 12-bus, modified IEEE 12-bus and IEEE 69-bus test systems and are shown in Figs. 2–4.

From Figs. 2–4, it can be observed that VSI is maximum at 9th bus (8th branch), 9th bus (8th branch), 61st bus (60th branch) for IEEE 12-bus, modified IEEE 12 bus and IEEE 69-bus test systems. Hence the optimal locations for DG placement are 9th bus, 61st bus for 12-bus, 69-bus systems. After identifying the potential nodes for DG placement, the search algorithm is used for optimal size of DG to achieve minimum network losses. By the Novel Power

Loss Sensitivity approach [27], the optimal locations obtained for DG placement are 9th bus, 61st bus for 12-bus and 69-bus systems respectively. By the Power Stability Index (PSI) method [18], the optimal locations obtained for DG placement are 9th bus, 61st bus for 12-bus, 69-bus systems respectively.

### Voltage stability margin (VSM)

Present day power systems are being operated closer to their stability limits due to economic and environmental constraints. Maintaining a stable and secure operation of a power system is therefore a very important and challenging issue. Voltage instability results from the inability of the system to provide the power requested by loads. The driving force for voltage instability is increased load. The voltage stability margin is a parameter that identifies the near collapse nodes. The node with small stability indices are called weak nodes and then should be reinforced by injecting reactive power. In the present analysis voltage stability margin is calculated for time variant realistic ZIP load model. Also the impact of load growth on voltage stability indices is determined. The impact of DG on voltage stability improvement has also determined. When DGs are optimally installed in distribution network, the bus voltages will increase and voltage security will enhance.

Voltage stability margin [28] is determined for each bus using Eq. (18) and the bus with minimum VSM is determined. VSM of each bus is a number between 0 and 1.

$$VSM(re(i)) = V(se(i))^4 - 4(P(i)x(i) - Q(I)r(i))^2 - 4(V(se(i))^2(P(i)r(i) + Q(i)x(i)) \quad \text{for } i = 1, 2, \dots, Nb \quad (18)$$

where  $P$  is the sum of the real power loads of all the nodes beyond each node, plus the real power load at each node itself, plus the sum of real power losses of all branches beyond each node.  $Q$  is the sum of the reactive power loads of all the nodes beyond each node, plus the reactive power load at each node itself, plus the sum of reactive power losses of all branches beyond each node.

### Cost of energy loss and cost of DG

The cost of energy losses and cost component of reactive power has been calculated based on the mathematical model represented as:

- (i) Cost of energy losses (CL): the annual cost of energy loss is given by

$$CL = (\text{Total Real power Loss}) * (E_c * T) \quad \$ \quad (19)$$

$E_c$ : energy rate (\$/kW h).

$T$ : time duration (h).

where

$$E_c = 0.06 \text{ \$/kW h}$$

$$T = 8760 \text{ h}$$

- (ii) Cost component of DG for real and reactive power

Cost characteristic of DG is selected as per the data available in [23]

$$C(Pdg) = a * Pdg^2 + b * Pdg + c \text{ \$/h} \quad (20)$$

Cost coefficients are taken as:  $a = 0$   $b = 20$   $c = 0.25$ .

Cost of reactive power supplied by DG is calculated based on maximum complex power supplied by DG as [29]

$$C(Qdg) = \left[ \text{Cost}(Sgmax) - \text{Cost} \left( \sqrt{Sgmax^2 - Qg^2} \right) \right] * k \text{ \$/h} \quad (21)$$

$$S_{gmax} = \frac{P_{gmax}}{\cos \theta}$$

$$P_{max} = 1.1 * P_g$$

$$k = 0.05 - 0.1$$

In this paper work, the value of factor  $k$  is taken as 0.1.

## Results and discussion

Based on the proposed VSI approach, DGs are placed for voltage profile improvement and to reduce total power losses. The results for IEEE 12-bus, modified IEEE 12-bus, IEEE-69 bus and 85-bus test systems have been obtained for voltage profile, total power losses, voltage stability margin profile, real and reactive power flow patterns, cost of energy loss, cost of real and reactive powers obtained from DG, and annual cost of energy loss savings, without and with installation of DGs. The results obtained with the proposed VSI method is also compared with existing PSI and Novel Power Loss Sensitivity methods on four test systems to demonstrate its effectiveness.

### Results for IEEE 12 bus test system using proposed VSI approach

The results have been obtained for IEEE 12 bus RDS with VSI method. The base MVA and base kV of the test system are:  $(MVA)_{Base} = 100$  MVA  $(kV)_{Base} = 11$  kV.

For 12 bus system without installation of DG, real and reactive power losses are 20.71353 kW and 8.041039 KVAR respectively. Real and reactive power from the substation is 455.7135 kW and 413.041 KVAR respectively. It is found that VSI is maximum at bus 9 as shown in Fig. 2. Therefore, it has been selected as a candidate node for DG allocation. Placing DG at 9th bus and varying the sizes of DG in steps, the variation of total real power loss with DG size is obtained. Real power loss variation with DG size at unity and lagging power factor is shown in Fig. 5. Total real power loss is obtained minimum with DG of 235 kW and 305 kVA at unity and 0.9 power factor lagging respectively. With installation of DG at bus 9 at unity power factor, the real and reactive power losses are 10.77397 kW and 4.125928 KVAR respectively. Also real and reactive power from the substation reduces to 210.774 kW and 409.1259 KVAR. With DG at 0.9 power factor lag, real and reactive power losses are 4.49371 kW and 1.634711 KVAR respectively. Real and reactive powers received from the substation obtained are 164.9937 kW and 273.6883 KVAR.

The voltage profile obtained with unity and lagging power factor is shown in Fig. 6. It is observed from the simulation results that the DG size obtained is higher at lagging power factor compared to the size obtained at unity power factor, however, the losses are found lower with DG at lagging power factor rather than DG at unity power factor. This is due the reason of reactive power

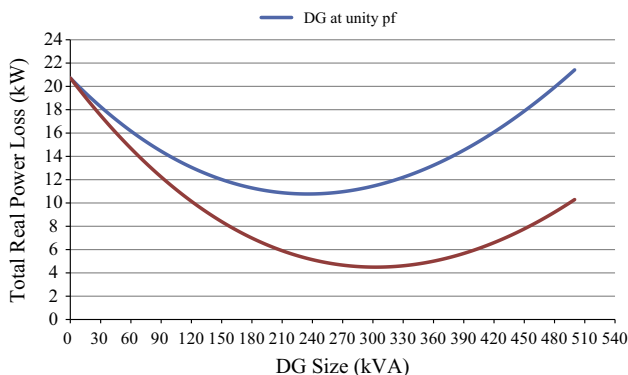


Fig. 5. Total power loss variation with DG size for IEEE 12 bus RDS.

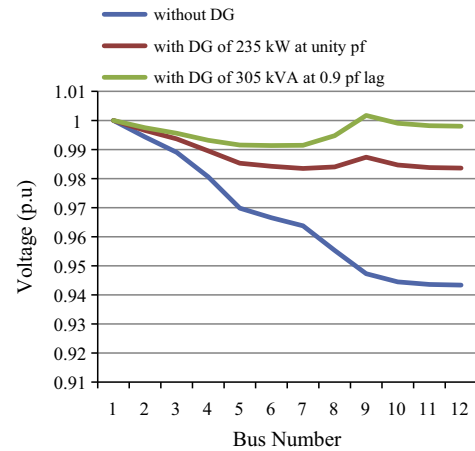


Fig. 6. Voltage profile for IEEE 12 bus RDS.

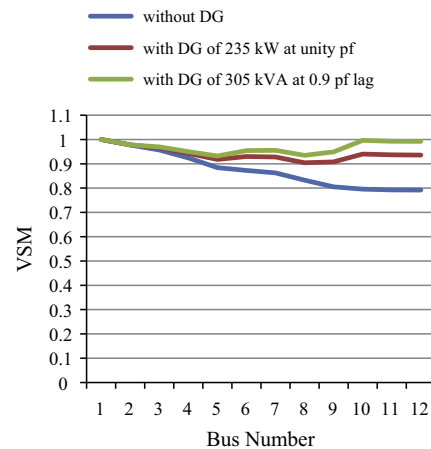


Fig. 7. VSM profile for IEEE 12 bus RDS.

available locally for the loads and thereby decreases in the reactive power taken from substation under DG operating at lagging power factor. The voltage profile also improves with DG at lagging power factor than DG at unity power factor and it is observed from Fig. 6. Minimum bus voltage is increased from 0.943354 p.u., to 0.983492 p.u., and 0.991382 p.u., with installation of DGs at unity and lagging power factor at 9th bus respectively. The minimum voltage obtained with DG at lagging power factor is better compared to the voltage obtained with DG at unity power factor. Fig. 7 is presented for investigation of Voltage Stability Margin (VSM) in two cases of before and after DG installation. As this figure shows, after optimal placement of DG, voltage stability margin is improved considerably at each node. The minimum VSM improves from 0.79195 to 0.90443, and 0.93227 with installation of DGs at unity and lagging power factor at 9th bus respectively.

The operation of the distribution system has changed from passive to active network with the integration of distributed generation sources at the various locations of the distribution system. The increased proliferation of these distributed generators has lead to changes in the characteristics of the network, with more variable and bidirectional active and reactive power flows. The real and reactive power flow patterns are depicted in Figs. 8 and 9. Due to reduced total power loss with installation of DGs, there is significant decrement in cost of energy loss. The cost of energy loss is reduced from \$ 10887.03, to \$ 5662.796 and \$ 2361.894, with

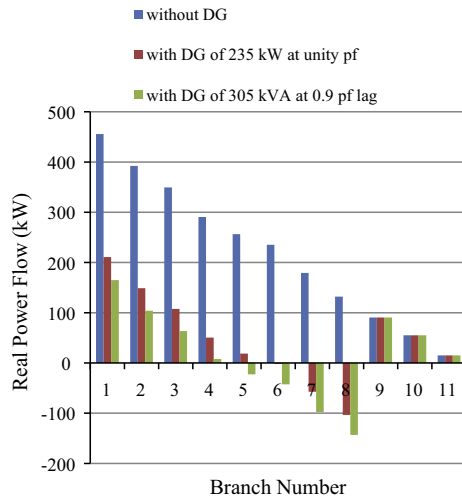


Fig. 8. Real power flow for IEEE 12 bus RDS.

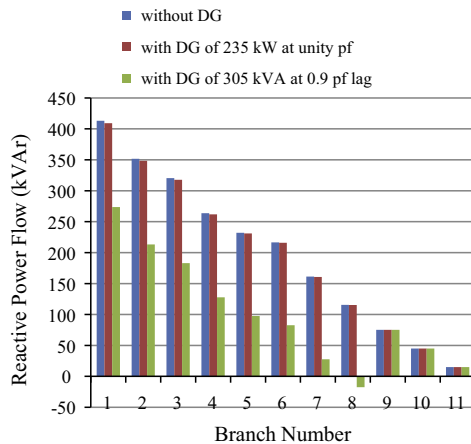


Fig. 9. Reactive power flow for IEEE 12 bus RDS.

installation of DG at unity and lagging power factor respectively. It results to annual savings of \$ 5224.2 and \$ 8525.1 with DGs at unity and lagging power factor respectively. Thus, it is essential to consider the reactive power available from DGs for its size calculations and its impact on total power losses reduction, voltage profile improvement, voltage stability margin improvement, and cost of energy savings. The summary of the results obtained

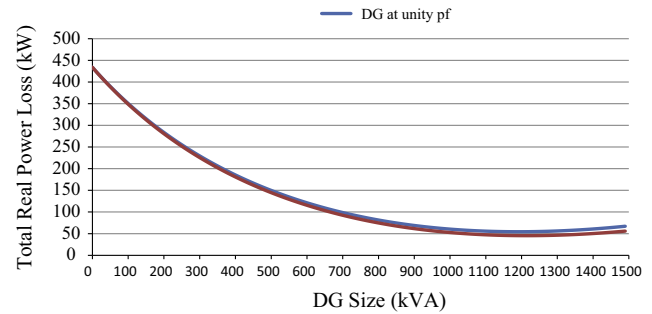


Fig. 10. Total power loss variation with DG size for modified IEEE 12 bus RDS.

without and with installation of DGs is given in Table 1 using VSI method. The results obtained with consideration of reactive power from DGs are better than the results obtained with DGs at unity power factor.

#### Results for modified IEEE 12 bus test system using proposed VSI approach

The results have been obtained for modified IEEE 12 bus RDS with VSI method. In modified 12-bus system, the active load on each bus is multiplied by factor 5 for better visualization of results, as the actual value of load is very small. The base MVA and base kV of the test system are:  $(MVA)_{Base} = 100$  MVA  $(kV)_{Base} = 11$  kV. For 12 bus system without installation of DG, real and reactive power losses are 434.0713 kW and 166.8206 KVar respectively. Real and reactive power from the substation is 2609.071 kW and 571.8206 KVar respectively. It is found that VSI is maximum at bus 9 as shown in Fig. 3. Therefore, it has been selected as a candidate node for DG allocation. Placing DG at 9th bus and varying the sizes of DG in steps, the variation of total real power loss with DG size is obtained. Real power loss variation with DG size at unity and lagging power factor is shown in Fig. 10. Total real power loss is obtained minimum with DG of 1190 kW and 1210 kVA at unity and 0.98 power factor lagging respectively. With installation of DG at bus 9 at unity power factor, the real and reactive power losses are reduced to 54.49956 kW and 19.33946 KVar respectively. Also real and reactive power from the substation reduces to 1039.5 kW and 424.3395 KVar. With DG at 0.98 power factor lag, real and reactive power losses are 45.2816 kW and 15.72981 KVar respectively. Real and reactive powers received from the substation obtained are 1034.482 kW and 179.9429 KVar.

The voltage profile obtained with unity and lagging power factor is shown in Fig. 11. It is observed from the simulation results that the DG size obtained is slightly higher at lagging power factor

Table 1  
Results for IEEE 12-bus RDS.

	Without DG	With DG at unity pf	With DG at 0.9 pf lag
DG location	–	9	9
DG size (kVA)	–	235	305
Total real power loss (kW)	20.71353	10.77397	4.49371
Total reactive power loss (KVar)	8.041039	4.125928	1.634711
Minimum bus voltage (p.u.) @bus	0.943354 @12	0.983492 @7	0.991382 @6
Minimum VSM @bus	0.79195 @12	0.90443 @8	0.93227 @5
Pload (kW)	435		
Qload (KVar)	405		
Pi/p (kW)	455.7135	210.774	164.9937
Qi/p (KVar)	413.041	409.1259	273.6883
Cost of PDG (\$/h)	–	4.95	5.74
Cost of QDG (\$/h)	–	–	0.05493
Cost of energy losses (\$)	10887.03	5662.796	2361.894
Savings in cost of energy losses (\$)	–	5224.2	8525.1



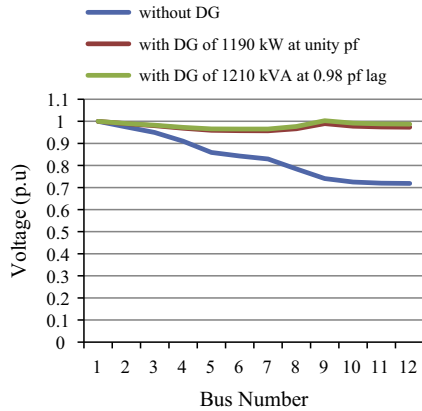


Fig. 11. Voltage profile for modified IEEE 12 bus RDS.

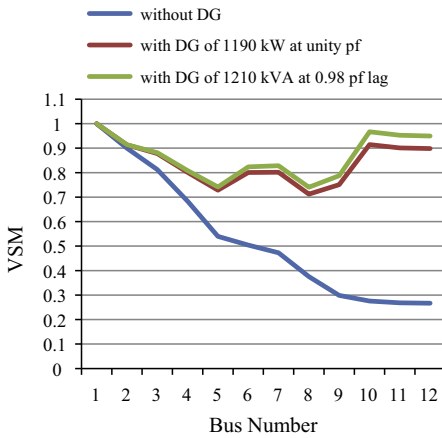


Fig. 12. VSM profile for modified IEEE 12 bus RDS.

compared to the size obtained at unity power factor, however, the losses are found slightly lower with DG at lagging power factor rather than DG at unity power factor. This is due the reason of reactive power available locally for the loads and thereby decreases in the reactive power taken from substation under DG operating at lagging power factor. The voltage profile also improves with DG at lagging power factor than DG at unity power factor and it is observed from Fig. 11. Minimum bus voltage is increased from 0.718656 p.u., to 0.956743 p.u., and 0.965004 p.u., with installation of DGs at unity and lagging power factor at 9th bus respectively. The minimum voltage obtained with DG at lagging power factor is slightly better compared to the voltage obtained with DG at unity power factor. Fig. 12 is presented for investigation of Voltage Stability Margin (VSM) in two cases of before and after DG installation. As this figure shows, after optimal placement of DG, voltage stability margin is improved considerably at each node. The minimum VSM improves from 0.26674 to 0.71244, and 0.74026 with installation of DGs at unity and lagging power factor at 9th bus respectively. Also the real and reactive power flows from sensing end to receiving end are shown in Figs. 13 and 14. It can be observed from these figures that, there is bidirectional power flows with installation of DGs in the distribution network. Due to reduced total power loss with installation of DGs, there is significant decrement in cost of energy loss. The cost of energy loss is reduced from \$ 228147.9, to \$ 28644.97 and \$ 23800.01, with installation of DG at unity and lagging power factor respectively. It results to annual savings of \$ 199502.91908 and \$ 204347.87720 with DGs at unity and lagging power factor

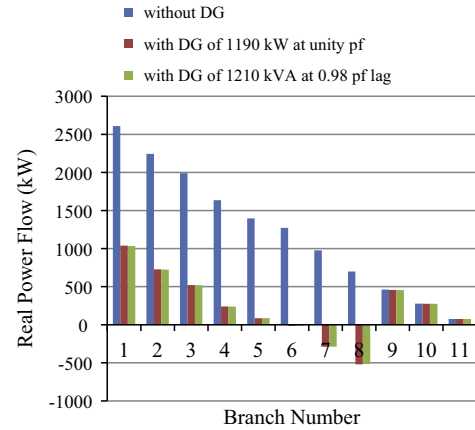


Fig. 13. Real power flow for modified IEEE 12 bus RDS.

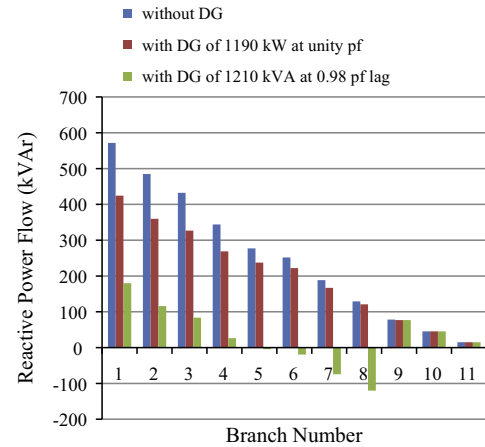


Fig. 14. Reactive power flow for modified IEEE 12 bus RDS.

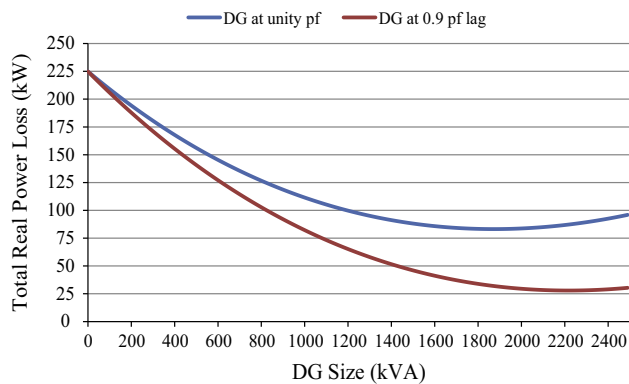
respectively. Thus, it is essential to consider the reactive power available from DGs for its size calculations and its impact on total power losses reduction, voltage profile improvement, voltage stability margin improvement, and cost of energy savings. The summary of the results obtained without and with installation of DGs is given in Table 2 using VSI method. The results obtained with consideration of reactive power from DGs are better than the results obtained with DGs at unity power factor.

#### Results for IEEE 69 bus test system using proposed VSI approach

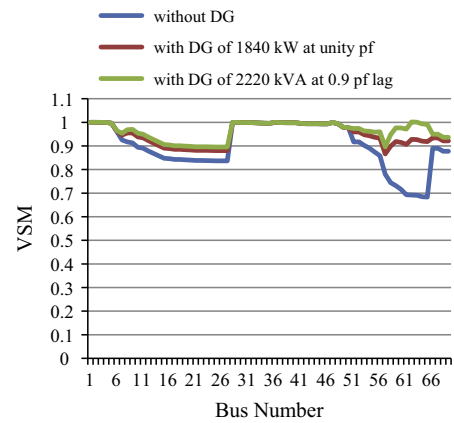
The results have been obtained for IEEE 69 bus RDS with VSI method. The base MVA and base kV of the test system are: (MVA)<sub>Base</sub> = 100 MVA (kV)<sub>Base</sub> = 12.66 kV. For 69 bus system without installation of DG, real and reactive power losses are 224.8688 kW and 102.1044 KVAR respectively. Real and reactive power from the substation is 4026.259 kW and 2795.704 KVAR respectively. It is found that VSI is maximum at bus 61 as shown in Fig. 3. Therefore, it has been selected as a candidate node for DG allocation. Placing DG at 61st bus and varying the sizes of DG in steps, the variation of total real power loss with DG size is obtained. Real power loss variation with DG size at unity and lagging power factor is shown in Fig. 15. Total real power loss is obtained minimum with DG of 1870 kW and 2220 kVA at unity and 0.9 power factor lagging respectively. With installation of DG at bus 61 at unity power factor, the real and reactive power losses are 83.13942 kW and 40.50004 KVAR respectively. Also real and

**Table 2**  
Results for modified IEEE 12-bus RDS.

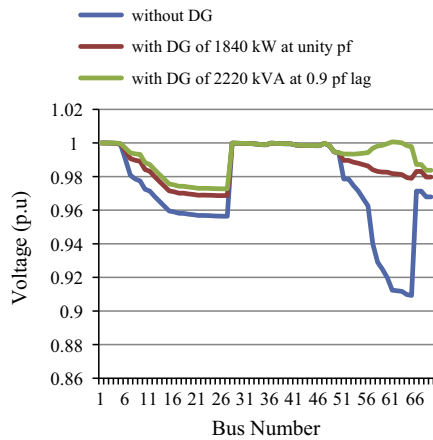
	Without DG	With DG at unity pf	With DG at 0.98 pf lag
DG location	–	9	9
DG size (kVA)	–	1190	1210
Total real power loss (kW)	434.0713	54.49956	45.2816
Total reactive power loss (KVar)	166.8206	19.33946	15.72981
Minimum bus voltage (p.u.) @bus	0.718656 @12	0.956743 @7	0.965004 @6
Minimum VSM @bus	0.26674 @12	0.71244 @8	0.74026 @8
Pload (kW)	2175		
Qload (KVar)	405		
Pi/p (kW)	2609.071	1039.5	1034.482
Qi/p (KVar)	571.8206	424.3395	179.9429
Cost of PDG (\$/h)	–	24.05	23.966
Cost of QDG (\$/h)	–	–	0.043922
Cost of energy losses (\$)	228147.9	28644.97	23800.01
Savings in cost of energy losses (\$)	–	199502.91908	204347.87720



**Fig. 15.** Total power loss variation with DG size for IEEE 69 bus RDS.



**Fig. 17.** VSM profile for IEEE 69 bus RDS.



**Fig. 16.** Voltage profile for IEEE 69 bus RDS.

reactive power from the substation reduces to 2014.529 kW and 2734.1 KVar. With DG at 0.9 power factor lag, real and reactive power losses are 27.89977 kW and 16.42448 KVar respectively. Real and reactive powers received from the substation obtained are 1831.29 kW and 1742.349 KVar.

The voltage profile obtained with unity and lagging power factor is shown in Fig. 16. It is observed from the simulation results that the DG size obtained is higher at lagging power factor compared to the size obtained at unity power factor, however, the losses are found lower with DG at lagging power factor rather than

DG at unity power factor. This is due the reason of reactive power available locally for the loads and thereby decreases in the reactive power taken from substation under DG operating at lagging power factor. The voltage profile also improves with DG at lagging power factor than DG at unity power factor and it is observed from Fig. 16. Minimum bus voltage is increased from 0.909202 p.u., to 0.968675 p.u., and 0.97273 p.u., with installation of DGs at unity and lagging power factor at 61st bus respectively. The minimum voltage obtained with DG at lagging power factor is better compared to the voltage obtained with DG at unity power factor. Fig. 17 is presented for investigation of Voltage Stability Margin (VSM) in two cases of before and after DG installation. As this figure shows, after optimal placement of DG, voltage stability margin is improved considerably at each node. The minimum VSM improves from 0.68335 to 0.86585, and 0.86585 with installation of DGs at unity and lagging power factor at 9th bus respectively. With installation of DGs, the real and reactive power flows in each branch are depicted in Figs. 18 and 19. Due to reduced total power loss with installation of DGs, there is significant decrement in cost of energy loss. The cost of energy loss is reduced from \$ 118191.1, to \$ 43698.08 and \$ 14664.12, with installation of DG at unity and lagging power factor respectively. It results to annual savings of \$ 74493 and \$ 204347.87720 with DGs at unity and lagging power factor respectively.

Thus, it is essential to consider the reactive power available from DGs for its size calculations and its impact on total power losses reduction, voltage profile improvement, voltage stability margin improvement, and cost of energy savings. The summary of the results obtained without and with installation of DGs is

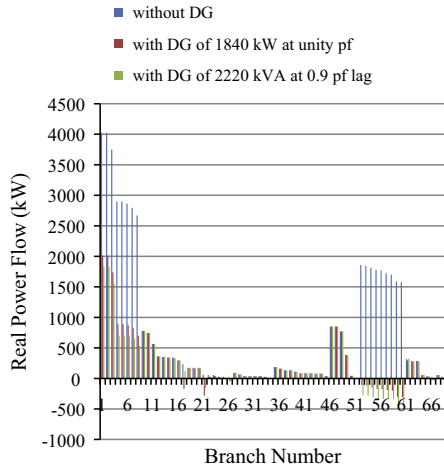


Fig. 18. Real power flow for IEEE 69 bus RDS.

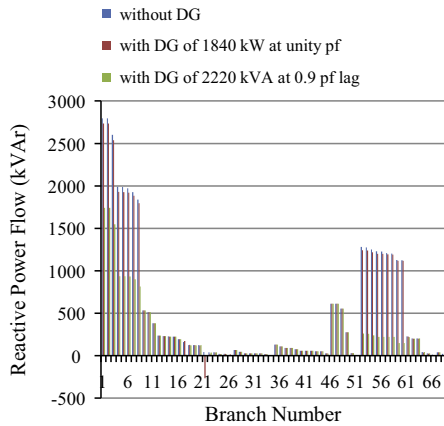


Fig. 19. Reactive power flow for IEEE 69 bus RDS.

given in Table 3 using VSI method. The results obtained with consideration of reactive power from DGs are better than the results obtained with DGs at unity power factor.

#### Optimal DG placement in RDS at optimal power factor considering load growth

In this section a summary of results obtained with optimal DG installation at optimal power factor [14] with consideration of load growth is presented incisively for three test systems. Load growth

in a system is a natural phenomenon. With the increase in load demand, system power loss and voltage drop increases. For future expansion and planning of the distribution systems, it is desirable that a system engineer must know the future estimate of the system solutions for planning and expansion or the efficient operation of distribution systems. In the present work, the load growth has been considered to study the impact on, optimal DG placement, power losses, voltage profile, real and reactive power requirement from the system along with the voltage stability margin, cost of energy loss, and cost of generated power obtained from DG with annual savings in cost of energy loss. In this paper work, load growth is modeled as

$$\text{Load}_i = \text{Load} \times (1 + r)^m \quad (22)$$

$r$  = annual growth rate,

$m$  = plan period up to which feeder can take the load

In this paper work load growth rate is taken as 7.5% and planning period as 5 years.

In this paper, the impact of optimal DG at optimal power factor is also determined for 12-bus, 69-bus and 85-bus systems in the presence of load growth. In order to minimize the total power losses, the operating power factor of DG ( $PF_{DG}$ ) should equals to power factor of combined load ( $PF_D$ ) on the feeder.

$$PF_D = \frac{P_D}{\sqrt{P_D^2 + Q_D^2}} \quad (23)$$

$$PF_{DG} = PF_D \quad (24)$$

The optimal power factors for DG placement are determined using Eq. (23) and are 0.73, 0.81 and 0.7 lagging power factors for 12-bus and 69-bus respectively. It can be observed from Table 7 that in the presence of load growth, the real and reactive powers drew from the substation increases since total load increases, power losses increase, and decline of voltage profile and voltage stability margin. Impact of DG operating at optimal power factors without consideration of load growth is also given in Tables 5 and 6 for comparison.

#### Comparison of results

The results obtained with proposed VSI method is compared with other existing sensitivity based approaches i.e., PSI and Novel Power Loss Sensitivity methods. This has been presented to evaluate the effectiveness of the proposed VSI method for optimal DG location as well as optimal sizes of DGs, improvement in voltage profile, improvement in voltage stability margin and reduction in total power losses, thereby the savings in cost of energy losses and cost of energy obtained from DGs. The comparison of results for the four test systems are given in Table 7.

Table 3  
Results for IEEE 69-bus RDS.

	Without DG	With DG at unity pf	With DG at 0.9 pf lag
DG Location	–	61	61
DG Size (kVA)	–	1870	2220
Total real power loss (kW)	224.8688	83.13942	27.89977
Total reactive power loss (KVar)	102.1044	40.50004	16.42448
Minimum bus voltage (p.u.) @bus	0.909202 @65	0.968675 @26	0.97273 @26
Minimum VSM @bus	0.68335 @65	0.86585 @57	0.86585 @26
Pload (kW)	3801.39		
Qload (KVar)	2693.6		
Pi/p (kW)	4026.259	2014.529	1831.29
Qi/p (KVar)	2795.704	2734.1	1742.349
Cost of PDG (\$/h)	–	37.65	40.21
Cost of QDG (\$/h)	–	–	0.39982
Cost of energy losses (\$)	118191.1	43698.08	14664.12
Savings in cost of energy losses (\$)	–	74493	204347.87720



**Table 5**

Results with installation of DG at optimal power factor for 12-bus RDS.

	Without DG	With DG at unity pf	With DG at 0.73 pf lag
<i>Results with installation of DG at optimal power factor for 12-bus RDS without load growth</i>			
DG location	–	9	9
DG size (kVA)	–	235	315
Total real power loss (kW)	20.71353	10.77397	3.158771
Total reactive power loss (KVAR)	8.041039	4.125928	1.107776
Minimum bus voltage (p.u.) @bus	0.943354 @12	0.983492 @7	0.99069 @7
Minimum VSI @bus	0.79195 @12	0.90443 @8	0.93105 @5
Pload (kW)	435		
Qload (KVAR)	405		
Pi/p (kW)	455.7135	210.774	208.21
Qi/p (KVAR)	413.041	409.1259	190.82
Cost of PDG (\$/h)	–	4.95	4.849
Cost of QDG (\$/h)	–	–	4.849
Cost of energy losses (\$)	10887.03	5662.796	1660.3
Savings in cost of energy losses (\$)	–	5224.2	9226.8
<i>Results with installation of DG at optimal power factor for 12-bus RDS with load growth</i>			
DG location	–	9	9
DG size (kVA)	–	345	455
Total real power loss (kW)	44.5927	22.59645	6.552925
Total reactive power loss (KVAR)	17.29762	8.636114	2.291374
Minimum bus voltage (p.u.) @bus	0.916647 @12	0.976475 @7	0.986723 @6
Minimum VSI @bus	0.70601 @12	0.86486 @8	0.90151 @5
Pload (kW)	624.4988		
Qload (KVAR)	581.4299		
Pi/p (kW)	669.0915	302.0952	298.9017
Qi/p (KVAR)	598.7275	590.066	272.7528
Cost of PDG (\$/h)	–	7.15	6.893
Cost of QDG (\$/h)	–	–	0.21666
Cost of energy losses (\$)	23437.92	11876.69	3444.217
Savings in cost of energy losses (\$)	–	11561	19994

**Table 6**

Results with installation of DG at optimal power factor for 69-bus RDS.

	Without DG	With DG at unity pf	With DG at 0.81 pf lag
<i>Results with installation of DG at optimal power factor for 69-bus RDS without load growth</i>			
DG location	–	61	61
DG size (kVA)	–	1870	2240
Total real power loss (kW)	224.8688	83.13942	23.124
Total reactive power loss (KVAR)	102.1044	40.50004	14.363
Minimum bus voltage (p.u.) @bus	0.909202 @65	0.968675 @26	0.97274 @26
Minimum VSI @bus	0.68335 @65	0.86585 @57	0.89517 @26
Pload (kW)	3801.39		
Qload (KVAR)	2693.6		
Pi/p (kW)	4026.259	2014.529	2010.1
Qi/p (KVAR)	2795.704	2734.1	1394.4
Cost of PDG (\$/h)	–	37.65	45.05
Cost of QDG (\$/h)	–	–	0.75871
Cost of energy losses (\$)	118191.1	43698.08	12154
Savings in cost of energy losses (\$)	–	74493	106040
<i>Results with installation of DG at optimal power factor for 69-bus RDS with load growth</i>			
DG location	–	61	61
DG size (kVA)	–	2720	3230
Total real power loss (kW)	505.9125	175.1943	48.45194
Total reactive power loss (KVAR)	228.5958	85.12873	29.92891
Minimum bus voltage (p.u.) @bus	0.863272 @65	0.954425 @26	0.960506 @26
Minimum VSI @bus	0.55538 @65	0.8083 @57	0.85093 @26
Pload (kW)	5457.387		
Qload (KVAR)	3867.011		
Pi/p (kW)	5963.299	2912.581	2889.539
Qi/p (KVAR)	4095.607	3952.14	2002.772
Cost of PDG (\$/h)	–	54.65	52.576
Cost of QDG (\$/h)	–	–	1.094
Cost of energy losses (\$)	265907.6	92082.12	25466.34
Savings in cost of energy losses (\$)	–	173830	240440

**Table 7**

Comparison of results test systems.

	PSI [18]	Novel power loss sensitivity [27]	Proposed VSI
<i>Comparison of results for IEEE 12-bus RDS</i>			
DG location	9	9	9
DG size (kW)	234.9	231.6988	235
Total real power loss (kW)	10.774	10.776	10.77397
Total reactive power loss (KVAR)	4.1261	4.1329	4.125928
Minimum bus voltage (p.u.) @bus	0.98348 @7	0.98307 @7	0.983492 @7
Minimum VSM @bus	0.9044 @8	0.90344 @8	0.90443 @8
Cost of PDG (\$/hr)	4.948	4.884	4.95
Cost of energy losses (\$)	5662.8	5664	5662.796
Savings in cost of energy losses (\$)	5224.2	5223	5224.2
<i>Comparison of results for modified IEEE 12-bus RDS</i>			
DG location	9	9	9
DG size (kW)	1200	1092.775	1190
Total power loss (kW)	54.51	56.053	54.49956
Total reactive power loss (KVAR)	19.254	20.765	19.33946
Minimum bus voltage (p.u.) @bus	0.95753 @7	0.94892 @7	0.956743 @7
Minimum VSM @bus	0.71494 @8	0.68796 @8	0.71244 @8
Cost of PDG (\$/hr)	24.25	22.105	24.05
Cost of energy losses (\$)	28651	29461	28644.97
Savings in cost of energy losses (\$)	199497.1797	198686.5119	199502.9190
<i>Comparison of results for IEEE 69-bus RDS</i>			
DG location	61	61	61
DG size (kW)	1863.1	1832.454	1870
Total real power loss (kW)	83.142	83.195	83.13942
Total reactive power loss (KVAR)	40.512	40.58	40.50004
Minimum bus voltage (p.u.) @bus	0.96864 @26	0.96846 @26	0.968675 @26
Minimum VSM @bus	0.86555 @57	0.86424 @57	0.86585 @57
Cost of PDG (\$/hr)	37.512	36.899	37.65
Cost of energy losses (\$)	43700	43727	43698.08
Savings in cost of energy losses (\$)	74491	74464	74493
<i>Comparison of results for 85-bus RDS</i>			
DG location	69	8	8
DG size (kW)	1000	2137.654	2140
Total real power loss (kW)	210.77	161.75	161.7424
Total reactive power loss (KVAR)	125.19	95.708	95.70095
Minimum bus voltage (p.u.) @bus	0.90025 @54	0.92694 @54	0.92699 @54
Minimum VSM @bus	0.65684 @54	0.73825 @54	0.73841 @54
Cost of PDG (\$/h)	20.25	43.003	43.05
Cost of energy losses (\$)	110780	85018	85011.8

It can be observed from Table 7 that the PSI method is not giving exact optimal DG location for 85-bus test system. Cost of energy losses for 85-bus test system is obtained higher with PSI method compared to the other method because of non optimal DG location. Looking into all aspects of power losses, voltage profile, cost component for losses, and cost component for power obtained from DGs, proposed VSI method is giving better results for all the test systems.

## Conclusions

In this paper a new voltage stability index (VSI) is developed for optimal DG placement in radial distribution systems. This paper presents comparison of proposed VSI method with two existing methods for optimal placement of DG for reduction of power losses and improvement in voltage profile. The study is carried out on two types of DG that are: DG operating at unity power factor and DG operating at 0.9 power factor lagging and DG operating at optimal power factor.

The results have been obtained with and without consideration of load growth for real and reactive power losses, voltage profile, voltage stability margin profile, real and reactive power flow patterns, cost of energy loss, cost component for real power and reactive power obtained from DGs, and annual cost of energy loss savings. It can be conclude that there is much reduction in real, reactive power losses, and improvement in voltage profile with DG at lagging power factor due to its reactive power supply to

the system. Therefore, DG operating at lagging power factor and supplying reactive power to the system is giving better results than DG at unity power factor. Especially, DG operating at optimal power factor i.e., combined load power factor results to obtain maximum benefits from DG in terms of voltage profile improvement, power loss reduction, improvement in voltage stability margin, reduction in cost of energy loss and savings in cost of energy loss. The proposed VSI method is giving better results for all the test systems. Proposed VSI will provide planning better locations for distributed generation sources and better management of real and reactive power deployment.

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